MATH 147 QUIZ 7 SOLUTIONS

1. Calculate the improper integral $\iint_D \ln\sqrt{x^2+y^2}\,dA$ for $D = 0 \le x^2+y^2 \le 1$. (5 Points)

   We make a polar substitution, with $x = r\cos(\theta), y = r\sin(\theta)$, giving us $x^2+y^2 = r^2$, with $0 \le r \le 1$. After doing the transformation, we integrate

   $$\int_0^{2\pi}\int_0^1 \ln(r)\cdot r\;dr\;d\theta.$$

   We note this is an improper integral, as $\ln(r)$ is undefined for $r = 0$. First, we calculate $\int_a^1 r\ln(r)dr$. We proceed via parts, with $u = \ln(r)$ and $dv = r\;dr$, so we have

   $$\int_a^1 r\ln(r)dr = \ln(r)\left(\frac{r^2}{2}\right)\Big|_a^1 - \int_a^1 \frac{r}{2} = \left[\ln(r)\frac{r^2}{2} - \frac{r^2}{4}\right]_a^1 = \frac{-1}{4} - \ln(a)\frac{a^2}{2} - \frac{a^2}{4}.$$

   Next, take the limit of the above as $a \to 0$. This gives $\int_0^1 r\ln(r)dr = \frac{-1}{4}$. Then, do the outer integral, and get

   $$\int_0^{2\pi}\int_0^1 \ln(r)\cdot r\;dr\;d\theta. = \int_0^{2\pi}\frac{-1}{4}\;d\theta = -\pi/2.$$

2. Set up $\iiint_B (xy+xz+yz)\;dV$ as an iterated integral, where B is given by

   $$B = \{(x,y,x)|0 \le x \le 1, -x^2 \le y \le x^2, - \le z \le 1\}.$$

   Do not calculate the resulting iterated integral. (5 points)

   This region is $y-$simple, so we integrate with respect to y first, and the order of the other two does not matter. This gives us

   $$\iiint_B (xy+xz+yz)\;dV = \int_0^1\int_0^1\int_{-x^2}^{x^2}(xy+xz+yz)\;dy\;dx\;dz.$$